By comparison with Table 2.8, the maximum ratio for this form of construction would be 5. Therefore the section satisfies lateral buckling requirements.

Deflection

Permissible deflection $\delta_p = 0.003 \times \text{span} = 0.003 \times 4250 = 12.75\,\text{mm}$

$$\text{Actual deflection } \delta_a = \delta_m + \delta_v = \frac{5}{384}\frac{WL^3}{EI} + \frac{19.2M}{AE}$$

$$= \frac{5}{384} \times \frac{4.5 \times 10^3 \times 4250^3}{8800 \times 33.3 \times 10^6} + \frac{19.2 \times 2.39 \times 10^6}{10 \times 10^3 \times 8800}$$

$$= 15.35 + 0.52 = 15.87\,\text{mm} > 12.75\,\text{mm}$$

Therefore the 50 mm × 200 mm joist is not adequate in deflection.

As a guide to choosing a suitable section for deflection, the I_{xx} required to satisfy bending deflection alone (ignoring shear deflection) can be calculated in relation to the permissible deflection. We have

$$\delta_m = \frac{5}{384}\frac{WL^3}{EI}$$

Thus for δ_m only, rewriting this equation and using $\delta_p = 12.75\,\text{mm}$,

$$I_{xx} \text{ required} = \frac{5}{384}\frac{WL^3}{E\delta_p}$$

$$= \frac{5}{384} \times \frac{4.5 \times 10^3 \times 4250^3}{8800 \times 12.75} = 40.09 \times 10^6\,\text{mm}^4$$

Comparing this with the I_{xx} properties for sawn joists given in Table 2.4 shows that the previous 50 mm × 200 mm joist has an I_{xx} of $33.3 \times 10^6\,\text{mm}^4$ and would not suffice. By inspection of the table a more reasonable section to check would be 50 mm × 225 mm with an I_{xx} of $47.5 \times 10^6\,\text{mm}^4$. Hence

$$\delta_a = \frac{5}{384} \times \frac{4.5 \times 10^3 \times 4250^3}{8800 \times 47.5 \times 10^6} + \frac{19.2 \times 2.39 \times 10^6}{11.3 \times 10^3 \times 8800}$$

$$= 10.76 + 0.46 = 11.22\,\text{mm} < 12.75\,\text{mm}$$

Therefore the 50 mm × 225 mm joist is adequate in deflection, and will also be adequate in bending.

It would perhaps be simpler in future to adopt this approach for determining a trial section at the beginning of the deflection check.

Shear unnotched

$$\text{Maximum shear } F_v = \text{reaction} = \frac{\text{UDL}}{2} = \frac{4.5}{2} = 2.25\,\text{kN} = 2.25 \times 10^3\,\text{N}$$

Grade shear stress parallel to grain (from Table 2.2) $r_g = 0.67\,\text{N/mm}^2$

Permissible shear stress $r_{adm} = r_g K_3 K_8 = 0.67 \times 1.25 \times 1.1 = 0.92\,\text{N/mm}^2$

$$\text{Applied shear stress } r_a = \frac{3}{2}\frac{F_v}{A} = \frac{3}{2} \times \frac{2.25 \times 10^3}{11.3 \times 10^3} = 0.3\,\text{N/mm}^2 < 0.92\,\text{N/mm}^2$$

Thus the section is adequate in shear unnotched.

Shear notched

Check the section with a 75 mm deep bottom edge notch at the bearing as illustrated in Figure 2.4. The permissible shear stress for a member notched in this manner must be multiplied by a further modification factor K_5. For bottom edge notches,

$$K_5 = \frac{h_e}{h} = \frac{150}{225} = 0.67$$

Hence

$$r_{adm} = r_g K_3 K_8 K_5 = 0.92 \times 0.67 = 0.62\,\text{N/mm}^2$$

$$r_a = \frac{3}{2}\frac{F_v}{bh_e} = \frac{3}{2} \times \frac{2.25 \times 10^3}{50 \times 150} = 0.45\,\text{N/mm}^3 < 0.62\,\text{N/mm}^2$$

Therefore the section is also adequate in shear when notched as shown in Figure 2.4.

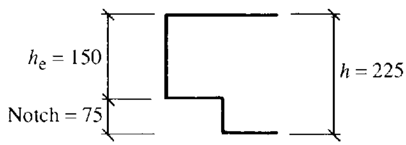

Figure 2.4 *Notched joist*

Bearing

Maximum bearing force F = reaction = 2.25×10^3 N

Assuming that the roof joists span on to a 100 mm wide wall plate, the bearing length will be 100 mm and hence the bearing area will be this length multiplied by the section breadth.

$$\text{Applied bearing stress } \sigma_{c,a,perp} = \frac{F}{\text{bearing area}} = \frac{2.25 \times 10^3}{100 \times 50} = 0.45\,\text{N/mm}^2$$

The grade bearing stress is the compression stress perpendicular to the grain from Table 2.2. This will be taken as the higher of the two values on the basis that wane will be specifically excluded at bearing positions. Therefore

Grade bearing stress $\sigma_{c,g,perp} = 2.2\,\text{N/mm}^2$

$$\begin{aligned}\text{Permissible bearing stress } \sigma_{c,adm,perp} &= \sigma_{c,g,perp} K_3 K_8 \\ &= 2.2 \times 1.25 \times 1.1 \\ &= 3.03\,\text{N/mm}^2 > 0.45\,\text{N/mm}^2\end{aligned}$$

The section is adequate in bearing

Joist self-weight

Finally, the load that was assumed for the joists can be verified now that the size is known and given that the timber weighs 540 kg/m^3: